Metal Combustion Efficiency Predictions for Low L^* Rocket Motors

Merrill K. King*
Atlantic Research Corporation, Alexandria, Virginia

Abstract

A MODEL for prediction of metal (aluminum or zirconium) combustion efficiency in nozzleless motors, including effects of residence time distributions due to particles emanating from different axial locations, particle agglomeration, two-dimensional flow patterns, particle lags, oxidizer depletion, and slip augmentation of particle burn rate, has been developed. With this model, extensive parametric studies have been converted into correlations for use in a code for analysis of nozzleless motor performance. In this code, these correlations have been combined with particle agglomeration correlations to permit calculation of metal combustion efficiency as a function of propellant, motor configuration, and operating conditions.

Contents

The objective of this analysis is calculation of the fraction of metal (aluminum or zirconium) burned in a rocket motor at any time during motor operation. Careful examination[1] indicates that existing efficiency codes are inadequate for analysis of nozzleless motors. Accordingly, a new code, removing deficiencies in previous codes, has been developed. This code is broken into three sections as described below.

First, a detailed trajectory analysis was developed for the calculation of fraction of metal burned as a function of five independent parameter groupings. Assumptions made to keep the analysis tractable while introducing minimal sacrifice in compuational accuracy include: 1) a cosine-law profile[2,3] for radial distribution of axial gas velocity was assumed; 2) radial variation of density was neglected in calculating radial velocity distributions; 3) collisions between particles were neglected; 4) continuum flow was assumed in the calculation of particle lags; 5) a simple empirical law was used to relate the mass transfer Nusselt number for oxidizer transport to particle Reynolds number; 6) particle combustion was assumed to be diffusion-limited; 7) each particle-gas pocket leaving the propellant surface; and 12) agglomerates were transport between them; 8) a straight bore port was assumed; 9) variation in the injection mass flux with axial location was neglected; 10) size distributions of metal particles leaving the propellant surface were assumed to be independent of axial location; 11) particles were assumed to begin burning upon leaving the propellant surface; and 2) agglomerates were assumed to burn by the same laws (determined from laboratory studies) as virgin particles.

Gas velocity distributions first were calculated in an uncoupled fashion, with subsequent Lagrangian calculation of trajectories of burning particles emanating from various axial locations. Details of the analysis are presented in Ref. 1. The time/cost of running this analysis directly coupled with a motor performance code is prohibitive. Accordingly, a minimum set of groups (Table 1) of independent parameters (motor geometry, operating condition, formulation) which uniquely define metal combustion efficiency was identified and a correlation of efficiency vs these groups was developed for each metal using the detailed model. For both metals, correlations of a "baseline" efficiency as a function of groups 1, 2, and 3 (with group 4 = 15.0 and group 5 = 5.0) were developed, with subsequent second-order correction for other values of these latter two groups. At any value of group 1, use of a normalizing parameter, GRP3BASE, allowed the baseline efficiencies to be collapsed into a single curve on a semilogarithmic plot of η_{baseline} vs group 3/GRP3BASE, where GRP3BASE is related to group 2 by a power law. Thus,

$$\eta_{\text{baseline}} = a + b\ell n\{\text{group3/GRP3BASE}\} + d\ell n^2\{\text{group3/GRP3BASE}\} \quad (1)$$

$$\text{GRP3BASE} = \exp\{A - B\ell n(\text{group2})\} \quad (2)$$

where A, B, a, b, and d are functions of group 1. In practice, A, B, a, b, and d are tabulated for several values of group 1 (Tables 2 and 3), Eqs. (1) and (2) are used to calculate η_{baseline} at tabulated values closely bracketing the actual value and logarithmic interpolation is used to calculate actual η_{baseline}.

Effects on efficiency of group 4 values other than 15.0 and group 5 values other than 5.0 are represented by the following correlations.

Group 4 Correction Formulas

Aluminum:

$$\eta/\eta_{\text{baseline}} = 1.0 + 0.64\,(1-\eta_{\text{baseline}})^{1.5}\,\ell n(\text{group 4}/15) \quad (3a)$$

Zirconium:

$$\eta/\eta_{\text{baseline}} = 1.0 + 0.55(1-\eta_{\text{baseline}})\ell n\,(\text{group 4}/15) \quad (3b)$$

Group 5 Correction Formulas

Aluminum:

$$\eta/\eta_{\text{baseline}} = \text{CFMAX for } 0.6 < \eta_b < 0.9$$
$$= \{1.0 + (\eta_b/0.6)(\text{CFMAX} - 1)\} \text{ for } \eta_b < 0.6$$
$$= \{1.0 + 10(1-\eta_b)(\text{CFMAX} - 1)\} \text{ for } \eta_b > 0.9 \quad (4a)$$

$$\text{CFMAX} = 0.939 + 0.305/(\text{group5}) \quad (5a)$$

Zirconium:

$$\eta/\eta_{\text{baseline}} = \text{CFMAX for } 0.6 < \eta_b < 0.8$$
$$= \{1.0 + (\eta_b/0.6)(\text{CFMAX} - 1)\} \text{ for } \eta_b < 0.6$$
$$= \{1.0 + 5(1-\eta_b)(\text{CFMAX} - 1)\} \text{ for } \eta_b > 0.8 \quad (4b)$$

$$\text{CFMAX} = 0.933 + 0.335/(\text{group 5}) \quad (5b)$$

For closure, the distribution of particle sizes leaving the surface as a function of propellant and motor parameters must be defined. Several options have been built into the overall metal combustion efficiency model. These various optional procedures are outlined in Ref. 1: only the "default" options, based on correlation of an extensive data base presented in Refs. 4-6, are discussed here. In each case, a number of "bins" with specified mean diameter of particles

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*Chief Scientist, Research and Technology Division. Member AIAA.

Table 1 Five correlating groups used for calculation of combustion efficiency of particles of a given size

	Aluminum	Zirconium
Group 1	$\frac{D_{po}^{1.5}\sqrt{T_{flame}/MW}}{r_{mult/Al}(a_{K,init})^{0.9}L(A_{bore}/A_{throat})}$	$\frac{D_{po}^{2}\sqrt{T_{flame}/MW}}{r_{mult,Zr}(Y_{ox,init})L(A_{bore}/A_{throat})}$
Group 2	$\frac{D_{po}(w_{total}/A_{bore})X_{gas}}{\mu_{gas}}$	Same
Group 3	$\frac{\rho_{particle}D_{po}\sqrt{T_{flame}/MW}}{(\dot{w}_{total}/A_{throat})LX_{gas}}$	Same
Group 4	L/R_{port}	Same
Group 5	$a_{K,init}/a_{K,final}$	$Y_{ox,init}/Y_{ox,final}$

$a_k = 100$ (moles $O_2 + CO_2 + H_2O + OH$)/total gas moles; A_{bore} = port cross-sectional area, ft^2; A_{throat} = throat area, ft^2; D_{po} = initial particle diameter, microns; L = grain length, ft; R_{port} = port radius, ft; r_{mult} = multipliers for laboratory burn rate constants; T_{flame} = flame temperature, R; X_{gas} = mass of gas/total mass; W_{total} = total motor flow rate, lbm/s; μ_{gas} = viscosity, lbm/ft s.

Table 2 Values of constants appearing in the aluminum particle combustion efficiency correlation equations

Group 1[a]	A	B	a	b	d
5	29.946	1.4411	.990	.00956	−.00247
10	31.551	1.5029	.968	.02514	−.00526
20	32.416	1.5074	.928	.04781	−.00907
40	32.396	1.4726	.859	.07357	−.01066
80	33.165	1.4440	.825	.0883	−.01288
200	32.791	1.3554	.740	.1113	−.01300
500	33.139	1.3345	.584	.1246	−.00747
1500	33.380	1.3121	.390	.1150	0
5000	33.600	1.290	.150	.1050	0

[a] micron$^{1.5}$R$^{0.5}$lbmol$^{0.5}$/lbm$^{0.5}$ft

Table 3 Values of constants appearing in the zirconium particle combustion efficiency correlation equations

Group 1[a]	A	B	a	b	d
2000	28.602	1.3596	1.000	.00139	−.00174
6000	30.563	1.4098	.990	.01241	−.00418
18000	31.659	1.4004	.961	.03967	−.01249
50000	32.318	1.3939	.865	.08633	−.01578
150000	32.838	1.3599	.720	.1313	−.01656
500000	33.719	1.3246	.580	.1280	−.00651
1500000	34.345	1.3013	.420	.1268	0
4500000	34.701	1.2707	.260	.09634	+.00965

[a] micron^{2}R$^{0.5}$lbmol$^{0.5}$/lbm$^{0.5}$ ft

and weight fraction of the total metal are set up. The fraction of metal in each "bin" that burns within the motor is then calculated, multiplied by the "bin" metal weight fraction, and summed over all the "bins" for the overall efficiency.

With aluminum, the fraction agglomerated (F_{AGL}) and the mass median agglomerate size (D_{AGL}) for a given virgin aluminum mass median particle size ($D_{V,Al}$) fraction of oxidizer appearing in the fine mode ($W_{OX,F}$), fine mode oxidizer diameter ($D_{OX,F}$), coarse mode oxidizer diameter ($D_{OX,C}$), pressure (P), and aluminum loading (W_{Al}) are calculated by:

1) If $\bar{D}_{V,Al} > 50\,\mu$ no agglomeration.

2) $F_{AGL} = F_{200}\{1.0 - A(P-200)\}$ $P<600$ psia

$= F_{200}\{1.0 - 400A)\}\{1.0 - B(P-600)\}$ $P>600$ psia

where

$A = 0.0014 - 0.020W_{OX,F}$ $W_{OX,F} < 0.05$

$= 0.0002 + 0.004W_{OX,F}$ $W_{OX,F} > 0.05$

$B = 0.0001 + 0.0065W_{OX,F}$ $W_{OX,F} < 0.05$

$= 0.0004375 - 0.00025W_{OX,F}$ $W_{OX,F} > 0.05$

$F_{200} = F_{10} + (W_{OX,F} - 0.10)(10)(F_{10} - F_O)$ $W_{OX,F} < 0.10$

$= F_{10} - W_{OX,F} + 0.10$ $W_{OX,F} > 0.10$

$F_{10} = \{-156 + 88\log_{10}D_{OX,C}\}(6/D_{OX,F})^{0.1}/100$

$F_0 = \{-115.4 + 58.7\log_{10}D_{OX,C}\}(6/D_{OX,F})^{0.1}/100$

3) If $F_{AGL} < 0$, $F_{AGL} = 0$.

4) $\bar{D}_{AGL} = 300.0F_{AGL}(W_{Al}/0.18)^{0.333}$.

5) If $\bar{D}_{AGL} < \bar{D}_{V,Al}$, $\bar{D}_{AGL} = \bar{D}_{V,Al}$.

Agglomerate and virgin fractions are each separated into six groups with the assumption that each fraction can be represented by a log-normal distribution with a standard deviation of 1.70 and diameters of particles at the 10, 30, 50, 70, 85, and 95% points on the cumulative mass probability curve for each fraction are calculated, with 20% of the mass in that fraction being assigned to each of the first four diameters and 10% to each of the last two diameters.

With zirconium, the default operation for calculating the degree of agglomeration is simpler, due to a shortage of data. In this case, a correlation was developed for the overall (mixed virgin and agglomerate particles) mass median diameter:

$D_{MIX,Zr} = D_{V,Zr}\{1.0 + 6.0(0.4 - W_{F,OX})^{2.2}\}$ $W_{F,OX} < 0.4$

$D_{MIX,Zr} = D_{V,Zr}$ $W_{F,OX} > 0.4$

In this case, the particles subsequently were split into 10 "bins" with midpoints at the 10, 30, 45, 55, 65, 75, 82.5, 87.5, 92.5, and 97.5 cumulative mass points.

References

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